The Status of Mathematics in Portugal in the 16th-17th Centuries

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Investigating the status of mathematics in Portugal in the 16th and 17th Centuries is a *sine qua non* condition to a better understanding of how the making, teaching and learning of science developed in the country. After all, there is a consensus that mathematics holds a special status in the realm of science in general. It is as antique as the most ancient types of knowledge and has travelled along history shaping scientific culture. Its uniqueness is tangible also in its bursting out of its borders and becoming an instrument of knowledge for many other fields of research.

Mathematics developed as an axiomatic system in Ancient Greece, and the making of such an organized body of knowledge was shaped by strong criticisms from opponents

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and detractors. These criticisms had indeed extraordinary impact on the development of the discipline; in the first place, they were not made by individuals, but by entire schools of thought, including sophists, sceptics or epicureans; secondly these critics objected to every single part of the mathematical discourse (principles, constructions, demonstrations, textual formulation, relationship with other fields of knowledge, relationship with the world); finally, an historical memoir of such criticisms remained until today. Although their constant presence, several pitch moments along history, corresponding to periods of intense revival, show their key role in the new trends and developments of mathematics. In particular, they dominated the 16th and 17th century's philosophy of mathematics. Many examples illustrate such cultural import from antiquity. The following should be enough to make my point: ancient authors such as Aristotle and Sirianus tell us that Aristippus denied any use to mathematics[1]; Cicero adds that Epicurus thought all mathematical knowledge to be false[2]. As late as the 17th century, Vossius tells us that Peter Ramus called the detractors of mathematics “Aristippeos” and “Epicureos”, tracing its citation back to classical culture[3]. Of course it is not the citation itself that is relevant, but the fact that upholders and detractors of mathematics still were considered to

[1] Metaphysics III, 2.

[2] Academicarum Quaestionum, IV, cap. 32.

[3] Gerardi Joan. *Vossii Operum tomus tertius Philologicus (Tractatus Philologicus de Rhetorica, de Poetica, de Artium et Scientiarum natura ac constitutione…*), (Amstelodami, 1697), p. 63-4.

share a common cultural background that goes a long way back to classical antiquity.

At the same time that mathematics evolved into a body of systematized knowledge, a general theory of science was produced. This simultaneous development of a particular kind of knowledge and its related meta-knowledge created a bias with unexpected historical importance. In the first place, it became hard to know which influenced the other or if there was convergence into a common project. In the second place each of them was always dealt with on the basis of the other. The standard formulation of the classical theory of science can be read in the Aristotelian *Posterior Analytics*. The standard formulation of mathematics as a system can be found in the *Elements* of Euclid. Not unexpectedly, pretty soon criticisms to mathematics started being made in the context of the Aristotelian model of science. When commenting on Euclid's *Elements*, book I, Proclus tells us that incompatibility between mathematics and the standard Aristotelian model of science was already stated by ancient authors like Amphinomus and refuted by others, like Geminus. Proclus himself upholds mathematics in the context of such model of science, using its phraseology and framing its requisites. It provides, for generations to come, the entourage in which to explain the scientific properties of mathematics [4].

[4] *In Euclidem*, p. 202. We cite Proclus according to the edition of G. Friedlein, *Procli Diadochi in primum Euclidis Elementorum librum comentarii*, (Hildesheim: Georg Olms, 1967).

Mathematical progress and the rediscovery of the *Posterior Analytics* reinforced the debate from the 12th century onwards. We can trace it in a modified form in the 14th Century Oxford *Calculatores*. At the same time, in Paris, John Buridan, Hugh of Traiecto and Albert of Saxony dealt with the topic. In the 16th century Italy, the humanist Alessandro Piccolomini strengthened the debate when publishing his *Commentary on the Certainty of Mathematics,* in 1547[5]. In this work, the Italian scholar brought together ancient lines of thought based on Aristotle, Plato or Euclid, along with the commentaries on their work (Proclus, Simplicius, Themistius, Eustratius, Ammonius, Philoponus, or Averrois, among others). In his gathering of arguments against mathematics he not only took profit from arguments continuously taught, but also from those newly discovered through the recovery of ancient authors. Recent studies have already identified other intervenient Italian authors, and their arguments on the subject[6].

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[5] The full title of the work is: Alessandri Piccolominei, In *Mechanicas Quaestiones Aristotelis. Paraphrasis paulo quidem plenior. Ad Nicolaum Ardinghellum, Cardinalem Amplissimum. Eiusdem Commentarium de Certitudine Mathematicarum: In quo de Resolutione, Diffinitione, et Demonstratione: necnon de matera, et in fine logicae facultatis, quamplura continentur ad rem ipsam, tum mathematicam, tum Logicam, maxime pertinentia* (Excussum Romae, apud Antonium Bladum Asulanum, 1547).

[6] Bibliography on the theme is huge, thus we remind only a few but important works: P. Galluzzi, "Il 'Platonismo' del tardo Cinquecento e la filosofia di Galileo", in *Ricerche sulla cultura dell'Italia moderna,* ed. Paola Zambelli (Bari: editore Laterza, 1973), pp. 37-79; Giulio Cesare Giacobbe, "Il *Commentarium De Certitudine Mathematicarum Disciplinarum* di Alessandro Piccolomini", *Physis*, 1972, *14*(2):162-193; *id.*, "Francesco Barozzi et la *Quaestio De Certitudine Mathematicarum"*, *Physis,* 1972, *14*(4):357-374; *id.*, "La riflessione metamatematica di Pietro Catena", *Physis,* 1973, *15*(2):178-196; *id.*, "Un gesuita progressista nella «Quaestio de Certitudine mathematicarum» rinascimentale: Benito

Considerable attention must be paid to Jesuits as agents of transmission of this debate, of which they made strong echo. The educational goal and the worldwide dimension of the Society enlarged the social impact of the debate. Moreover, forty years after Piccolomini had published his treatise, Jesuits were discussing the plan of studies (*Ratio Studiorum*) for their colleges, and were in need of defining the curricular place of mathematics in the context of a religious education. The debate was a sign of deep social and corporative tensions, since each group of teachers (philosophers and mathematicians) tried to enforce its own influence within the educational system. Among defendants of mathematics, one can find the most important Jesuit mathematical authority, Father Cristopher Clavius, who, besides producing several documents about how to promote the study of mathematics within the Society[7], and arguing for the certainty of mathematics in his works[8], struggled for the

Pereyra", *Physis,* 1977, *19*(1):51-86; N. Jardine, "The Epistemology of the Sciences", in *The Cambridge History of Renaissance Philosophy*, eds. C. B. Schmitt, Q.R.D. Skinner and E. Kessler (Cambridge University Press, 1988), pp. 685-711; Paolo Mancoso, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford University Press, 1996); Antonella Romano, *La contre-réforme mathématique – constituition et diffusion d'une culture mathématique jésuite à la renaissance (1540-1640)*, (Rome: École Française de Rome, 1999); Anna De Pace, *Le Matematiche e il mondo – Ricerche su un dibattito in Italia nella seconda metà del Cinquecento* (Milano: Francoangeli, 1993).

7 At least four documents are relevant: "Ordo seruandus in addiscendis disciplinis mathematicis", "Modus quo disciplinae mathematicae in scholis Societatis possent promoueri", "De re mathematica instructio", "Oratio de modo promouendi in Societate studia linguarum politioresque litteras ac mathematicas". All documents can be found in Ladislau S.I. Lukács, ed., *Monumenta Paedagogica Societatis Iesu*, vol. 7, (Romae, Institutum Historicum Societatis Iesu, 1992), pp. 119-122.

8 Cristophori Clavii Bambergensis e Societate Iesu *Operum Mathematicorum Tomus Primus*, (Moguntiae, Sumptibus Antonii Hierat, excudebat Reinhardus Eltz, Anno MDCXI), p. 5.

establishment of an Academy of Mathematics in the *Collegio Romano*, thus providing institutional foothold for investigation on mathematics. This mathematical project, however, faced ferocious opposition from Jesuit professors of philosophy. The most authoritative among these was Benito Pereira, a teacher of philosophy at the *Collegio Romano*, who proposed a more radical thesis than the one by Piccolomini, denying mathematics any value at all as a science[9].

The growing number of students and teachers of mathematics and the developments of both pure and applied mathematics increased the reflection upon its scientific status. The main topic discussed included knowing if mathematics fitted the Aristotelian model of science, and this implied further discussion on the status of mathematical demonstrations. One of the questions asked was if mathematical demonstrations were causal, and if mathematical principles and premises were like to fulfil all Aristotelian requisites, that is, if they were true, primary, immediate, more known, prior and causes of the conclusions[10].

Although the importance of this topic in the rise of modern science is known, it has been only partially studied.

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9 Benedictii Pererii Societatis Iesu *De communibus omnium rerum naturalium principiis et affectionibus libri quindecim qui plurimum conferunt, ad eos octo libros Aristotelis, qui de Physico auditu inscribuntur, intelligendos. Adiecti sunt huic operi tres indices, unus capitum singulorum librorum; Alter Quaestionum; Tertius rerum. Omnia uero in hac quarta editione denuo sunt diligentius recognita, et emendata. Cum priuilegio, et facultate superiorum,* (Romae, Ex officina Iacobi Tornerii et Iacobi Biricchiae, MDLXXXVI; the first edition dates back to 1576), book 1, chapter XII (Scientiam speculatiuam non dici uniuoce de Mathematicis disciplinis et aliis, quoniam doctrina Mathematica non est proprie scientia) pp. 26-28.

10 The definition of scientific knowledge and the properties of the premises of a science can be found in Aristotle's *Posterior Analytics* II, 71b9-72b5.

Attention has been paid to printed works and not yet to manuscript sources. Moreover, little work has been done in contexts other than the Italian, French and English one. There is a need for a deeper study in specific local contexts so that we can have a European perspective on the subject.

My investigation tried to detect echoes of the debate in Portugal. First there was the need to set a corpus of texts to search into. Secondly, the debate being so linked to mathematical practice, there was a need to know who did mathematics in Portugal in the period of the scientific revolution, what level of mathematics were Portuguese able to achieve and at what extend there was Portuguese participation in the making of European mathematics. Finally, a twofold investigation was imposed: the study of the debate among scholars in Portugal, and among Portuguese scholars in Paris.

I concentrated on the set of manuscripts that can be found at the National Library (Lisbon). The collection of manuscripts at the National Library (Lisbon) is so vast a collection (the largest in a Portuguese library) that it gives us a representative view on how the theory of science and other logic topics were taught in the classroom, providing useful epistemological, social, geographical and chronological information. Within that set, we restricted ourselves to lessons of logic, which contain the study of the Aristotelian theory of science. Special attention was paid to Jesuits, because of their importance in the Portuguese educational system. Although several official documents were investigated, my main concern was to find out what they usually taught in their courses of Philosophy. Happily, I found out that Jesuit teachers of philosophy usually included a chapter about the scientific status of mathematics in their lessons of logic, in a section often named *De Scientia*. When dealing with the status of mathematics in their courses, many teachers followed closely the contents of the *Commentarii Collegii Conimbricensis e Societate Iesu In Vniuersam Dialecticam Aristotelis Stagiritae*, published in 1606. On the other hand, these influential *commentarii* were based in lessons taught before that date; so, it will not be wrong to say that they presented, in the context of a general theory of science, a printed edition of widespread arguments against mathematics usually taught by Jesuit philosophers in the 16^{th} and 17^{th} centuries.

Two theses were of particular interest despite their traditional appearance: the first one insisted that mathematics should be considered a science *absolute, communiter* or *lato modo,* but not if considered in the proper sense, that is, *proprie,* or, in other words, in the Aristotelian sense, a thesis that might lead to consider mathematics in the context of other scientific models; the second one supported that only pure mathematics was not to be considered a true Aristotelian science, thus allowing a higher status to mathematical physics. Still, the author of the *commentarii*, Sebastião do Couto, strongly believed science should be considered *proprie*, and made his thinking into actions when he forbade the printing of lessons taught in Portugal by a colleague of his, Cristophoro Borri, because of his upholding mathematics. The *Dialectica Conimbricensis* gathered views against mathematics until 1606 and represented the late scholastic final outcome of fifty years of teaching and reasoning on the topic. All arguments presented against mathematics still focused in some of the postulates of the Aristotelian model of science unlike the later *Logic of Port Royal*, also a milestone in the story of mathematical criticisms, which goes a step forward in incorporating other traditions. The theses presented in these commentaries prevailed in the Portuguese context until the end of the 17th Century, when General Tyrso Gonzalez ordered a reformation of the mathematical teaching in Portugal, forbidding philosophers from attacking mathematics.

As to Portuguese Jesuit mathematicians and mathematical practice in Portuguese Jesuit schools, one is struck by the scarce information on the subject. Only recently (and partially), has some light been thrown onto the subject. There was a need again to go into the manuscript sources. Although mathematics was indeed considered a minor subject of teaching, the Jesuit school of Santo Antão, in Lisbon, offered a course on mathematics continuously since 1590 until Jesuits were expelled from Portugal in 1759. The relevance of this school does not end in its unique course on mathematics but also in the fact that its first teacher of mathematics, João Delgado, had been a former disciple of Cristopher Clavius, above mentioned. João Delgado became aware of the debate via Clavius' Academy of Mathematics, and was the first Portuguese mathematician that expressly addressed the subject in his classes, thus providing evidence that there were also Portuguese scholars who contributed to the debate. There is further evidence of a programmatic and systematic defence of mathematics in lectures of successive teachers of that school during the 17th Century. One can see it in the first chapters of the courses offered by Giovanni Paolo Lembo, Cristophoro Borri and others of unknown authorship[11].

One second group of Portuguese intellectuals aware of the problematic about the status of mathematics was located in Paris. Three cases are worth mentioning:

- one of the most interesting episodes of the University of Paris that somehow is linked to Portuguese culture is that of the anti-Aristotelian libellous written by Pierre de la Ramée, entitled *Aristotelicae Animadversiones* (1543). When referring to the *Posterior Analytics,* P. de la Ramée included a strong criticism against the scientific status of mathematics. Most relevant is the fact that it was António de Gouveia, an important Portuguese scholar connected to the college of Santa Bárbara (Paris), the responsible for the first printed reply (*Pro Aristotele responsio aduersus Petri Rami calumnias*, 1543). He was also chosen for a disputation on the subject with Pierre de la Ramée in the presence of the king himself. Defence of the *Posterior Analytics* involved, given the nature of the opponent's attack, the endorsement of mathematics in the context of the Aristotelian model of science. The interest in the episode increases when one realizes the important role Gouveia played in the Portuguese educational system;

[11] The authors cited, their biography and works are listed in Ugo Baldini, "L'insegnamento della matematica nel Collegio di S. Antão a Lisbona, 1590-1640", in *Colóquio Internacional a Companhia de Jesus e a missionação no Oriente* (Lisboa: Brotéria / Fundação Oriente, 2000), pp. 275-310. Among the texts of unknown authorship, the reader is simply referred to the manuscript BN cod. 2260, not cited in Baldini's paper.

- a second case is that of Diogo de Sá who criticized the renowned Portuguese mathematician Pedro Nunes (*De Navigatione libri tres*..., Paris, 1549). Significantly, a part of his work, published in Paris, consisted of a non mathematical criticism; instead, Diogo de Sá criticised the fact that, as mathematician, Pedro Nunes could only aim at a partial study of nature, because of mathematics' failure to meet the requirements of a true science. In this way, Diogo de Sá inscribed his work in the context of the debate on the certainty of mathematics. This example also clearly indicates the need for investigating the men responsible for teaching mathematics in the Portuguese Court (a group that included names like Pedro Nunes and his substitute Domingos Peres).

- a third Portuguese scholar aware of the debate was Francisco Sanches, a prominent professor at the University of Paris, who exposed his opinion on mathematics in a letter written to Cristoph Clavius, citing some topics of the debate[12].

Evidence thus supports the claim that theses against mathematics prevailed among Portuguese philosophers, both in and outside Portugal. Nevertheless, the scarce number of Portuguese mathematicians was not enough to counter balance the controversy because they were unable to provide universities with the necessary human resources for teaching mathematics, as well as to create institutional foothold for developing mathematics.

[12] The most recent edition of this letter can be found in U. Baldini and P.D. Napolitani, *Cristoph Clavius: corrispondenza* (Dipartimento di Matematica-Università di Pisa, 1992) vol. 6.1, pp. 185-194 and vol. 6.2, pp. 110-116.

To sum up, my work tries to understand the different views on how to relate mathematics and other fields of knowledge among 16th and 17th mathematicians and philosophers. Another perspective I intend to explore is the ways in which institutions assimilate philosophical and social views on the status of the mathematics and solve tensions within their curricula organization. The overall result intends to provide the more general context of humanistic reception and approaches to classical culture. The main topic addressed is the debate on the status of mathematical demonstrations (the *Quaestio de Certitudine Mathematicarum*) and its reception in Portugal. To make such study possible, there was a need for studying ancient mathematics (mainly Euclid, and Proclus' *Commentary on the first book of Euclid's* Elements) and its renaissance reception; ancient philosophy of mathematics (mainly Plato, Aristotle and their commentators) and again its renaissance reception. Stress must be put in the convergence of history of mathematics and a broader understanding of Classical Culture and its modern interpretation, which includes clarifying the Greek mathematical tradition up to the 18th century; the history of institutions and libraries; the transmission of knowledge and circulation of new scientific ideas in Portugal to the end of the 17th century.